

Controlling Statistical Properties of a Cooper Pair Box Interacting with a Nanomechanical Resonator

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Abstract

We investigate the quantum entropy, its power spectrum, and the excitation inversion of a Cooper pair box interacting with a nanomechanical resonator, the first initially prepared in its excited state, the second prepared in a “cat”-state. The method uses the Jaynes-Cummings model with damping, with different decay rates of the Cooper pair box and distinct detuning conditions, including time dependent detunings. Concerning the entropy, it is found that the time dependent detuning turns the entanglement more stable in comparison with previous results in literature. With respect to the Cooper pair box excitation inversion, while the presence of detuning destroys the its collapses and revivals, it is shown that with a convenient time dependent detuning one recovers such events in a nice way.

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1. Introduction

In the last years there has been a great interest in the production of new nonclassical states of the quantized electromagnetic field, one of the interesting topics of Quantum Optics. Despite the field quantization in 1925, quantum optical effects were observed only seven decades after, the first of them being the antibunching effect, as predicted by Carmichael and Walls in 1976 [1], experimentally confirmed by Kimble, M. Dagenais and L. Mandel [3] in 1977. A second nonclassical effect was observed in 1985 by Slusher et al. [4], theoretically anticipated by Stoler et al. [5] in 1970. A third one, the oscillations in the photon statistical distribution, was observed in 1987 by Rempe et al. [6]. Since then, various nonclassical states of the quantized electromagnetic field were studied, including their practical realization in laboratories in different systems - one of them being the famous Schrödinger “cat” state, its generation being suggested by Yurke and Stoler [7], Davidovich et al. [8], etc; its first experimental observation was obtained by the group of Haroche [9]. More recently, the community became aware of the first experimental observation of the decoherence of the Schrödinger “cat” state, in both realms of optical [10] and atomic physics [11], and constituting the first observation of the passage through the frontier that separates the quantum and classical physics. After that, another interesting topic emerged, as the quantum teleportation of states, first suggested by Bernnett et al. [12], based on the nonlocal character of quantum mechanics and contextualized by the EPR entangled states [13]. This somewhat “bizarre” effect was first observed experimentally in 1997, by the group of Zeilinger [14], concerning the teleportation of a single photon state; the effect was later extended for atomic states and also for a huge quantity of photons [15]. Then, several publications in this line appeared in the literature [16, 17, 18, 19].

Besides the nonclassical effects of light field states, many researchers became interested in the study of new states and new effects they could exhibit, mainly concerning with their potential applications [20]. Then, it became also

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interesting the study of various schemes for the generation of nonclassical light states [21, 22, 23]. To this end, two lines of study emerged: (i) when the issue is concerned with a state of a stationary field, inside a high-Q microwave cavity; (ii) when concerning with a traveling field, either throughout the free space or a medium (optical fibers, beam splitters, prisms, etc.). In both cases various proposals appeared in the literature [24, 25, 26, 27]. The extension of these investigations for atomic systems has been also implemented. In this case the system no longer concerns with traveling fields or a field trapped inside a high-Q cavity; instead, it consists of atoms either inside or crossing a cavity, including atoms inside a magneto-optical trap [28, 29].

When focusing either the field or the atomic case the theoretical strategy starts from a Hamiltonian describing the atom-field system, traditionally treated via the Jaynes-Cummings model and the atom-field coupling usually considered as a constant parameter. Comparatively, the number of such works in the literature is very small when one considers the atom-field coupling and/or atomic frequency as a time dependent parameter [30, 31, 32, 33, 34], including the case of time dependent amplitude [35]. Nevertheless, this scenario is also relevant; for example, the state of two qubits (qubits stand for quantum bits) with a desired degree of entanglement can be generated via a time dependent atom-field coupling [36]; such coupling can modify the dynamical properties of the atom and the field, with transitions that involve a large number of photons [37]. In general, these studies are simplified by neglecting the atomic decay from an excited level. Theoretical treatments taking into account this complication of the real world also employs the Jaynes-Cummings model. In these case, as expected, one finds decoherence of the state describing the system, since the presence of dissipation destroys the state of a system as time flows.

Here, taking advantage of what we have learned on the atom-field interaction, we will study an advantageous system in practice (due to its rapid response and better controllability [38]) by considering a nanomechanical resonator (NR) interacting with a Cooper pair box (CPB). This nanodevice has its own interest since its macroscopic nature and peculiar effects of low-frequency noise in the solid-state impose obstacles requiring more careful studies than a mere translation from quantum optics. It has been explored in the study of quantum nondemolition measurements [39, 40], in the study of decoherence of nonclassical states, as Fock states and superposition or entangled states describing mesoscopic systems [41], etc. The fast advance in the technique of fabrication in nanotechnology implied great interest in the study of the NR system in view of its potential modern applications, as a sensor, largely used in various domains, as in biology, astronomy, quantum computation, and more recently in quantum information [42] to implement the quantum qubit [43] and in the production of nonclassical states, e.g.: Fock states [44], Schrödinger's "cat" states [45], squeezed states [46], clusters states [47], etc. In particular, when accompanied by superconducting charge qubits, the NR has been used to prepare entangled states [48]. Zhou et al. [46] have proposed a scheme to prepare squeezed states using a NR coupled to a CPB qubit; in this proposal the NR-CPB coupling is under an external control while the connection between these two interacting subsystems play an important role in quantum computation. Such a control is achieved via convenient change of system parameters, which can set "on" and "off" the interaction between the NR and the CPB, on demand.

One of the desired goals in this report is to verify the behavior and properties of an entangled state describing the CPB-NR system, via the Jaynes-Cummings model, by considering the energy dissipation in the CPB during its transitions from an excited level to a ground state. Another target is to verify if, and in which way, the time dependence of the CPB-NR coupling modifies the dynamical properties of the state describing a subsystem. We will also study the time evolution of the quantum entropy and its power spectrum, as well as the CPB excitation inversion. There are some evidences of entropy production, including the fact that the power spectrum of stationary systems and subsystems can be used as dynamical criteria for quantum chaos [49, 50]. For the entropy power spectrum, such criteria embody those already discussed in the literature concerned with fixed parameters. Then, it seems adequate to look at the various characteristics of the entropy to formulate a reasonable and sufficient universal dynamical criterium for the quantum chaos. The degree of entanglement, represented by the entropy in certain circumstances, has also shown itself being sensible to the presence of a classical chaos [51, 52].

2. Model hamiltonian for the CPB-NR system

There exist in the literature a large number of devices using the SQUID-base, where the CPB charge qubit consists of two superconducting Josephson junctions in a loop. In the present model a CPB is coupled to a NR as shown in Fig. (1); the scheme is inspired in the works by Jie-Qiao Liao et al. [43] and Zhou et al. [46] where we have substituted each Josephson junction by two of them. This creates a new configuration including a third loop. A

superconducting CPB charge qubit is adjusted via a voltage V_1 at the system input and a capacitance C_1 . We want the scheme attaining an efficient tunneling effect for the Josephson energy. In Fig.(1) we observe three loops: one great loop between two small ones. This makes it easier controlling the external parameters of the system since the control mechanism includes the input voltage V_1 plus three external fluxes $\Phi(\ell)$, $\Phi(r)$ and $\Phi_e(t)$. In this way one can induce small neighboring loops. The great loop contains the NR and its effective area in the center of the apparatus changes as the NR oscillates, which creates an external flux $\Phi_e(t)$ that provides the CPB-NR coupling to the system. In this

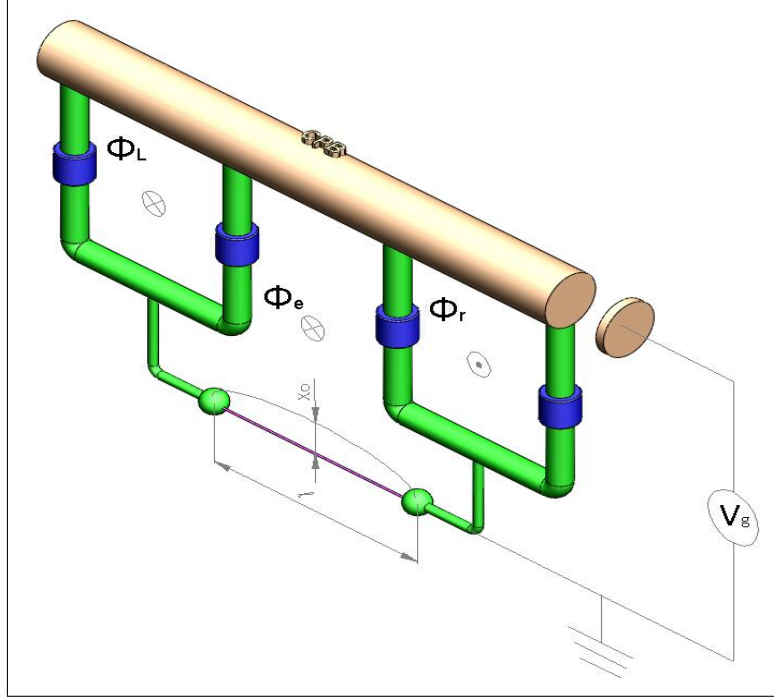


Figure 1: Model for the CPB-NMR coupling.

work we will assume the four Josephson junctions being identical, with the same Josephson energy E_J^0 , the same being assumed for the external fluxes $\Phi(\ell)$ and $\Phi(r)$, i.e., with same magnitude, but opposite sign: $\Phi(\ell) = -\Phi(r) = \Phi(x)$. In this way, we can write the Hamiltonian describing the entire system as

$$\hat{H} = \omega \hat{a}^\dagger \hat{a} + 4E_c \left(N_g - \frac{1}{2} \right) \hat{\sigma}_z - 4E_J^0 \cos\left(\frac{\pi\Phi_x}{\Phi_0}\right) \cos\left(\frac{\pi\Phi_e}{\Phi_0}\right) \hat{\sigma}_x, \quad (1)$$

where $\hat{a}^\dagger(\hat{a})$ is the creation (annihilation) operator for the excitation in the NR, corresponding with the frequency ω and mass m ; E_J^0 and E_c are respectively the energy of each Josephson junction and the charge energy of a single electron; C_1 and C_J^0 stand for the input capacitance and the capacitance of each Josephson tunnel, respectively. $\Phi_0 = h/2e$ is the quantum flux and $N_1 = C_1 V_1 / 2e$ is the charge number in the input with the input voltage V_1 . We have used the Pauli matrices to describe our system operators, where the states $|g\rangle$ and $|e\rangle$ (or 0 and 1) represent the number of extra Cooper pairs in the superconducting island. We have: $\hat{\sigma}_z = |g\rangle\langle g| - |e\rangle\langle e|$, $\hat{\sigma}_x = |g\rangle\langle e| - |e\rangle\langle g|$ and $E_C = e^2 / (C_1 + 4C_J^0)$.

The magnetic flux can be written as the sum of two terms,

$$\Phi_e = \Phi_b + B\ell\hat{x}, \quad (2)$$

where the first term Φ_b is the induced flux, corresponding to the equilibrium position of the NR and the second term describes the contribution due to the vibration of the NR; B represents the magnetic field created in the loop. We have assumed the displacement \hat{x} described as $\hat{x} = x_0(\hat{a}^\dagger + \hat{a})$, where $x_0 = \sqrt{m\omega/2}$ is the amplitude of the oscillation.

Substituting the Eq.(2) in Eq.(1) and controlling the flux Φ_b we can adjust $\cos\left(\frac{\pi\Phi_b}{\Phi_0}\right) = 0$ to obtain

$$\hat{H} = \omega \hat{a}^\dagger \hat{a} + 4E_c \left(N_g - \frac{1}{2}\right) \hat{\sigma}_z - 4E_J \cos\left(\frac{\pi\Phi_x}{\Phi_0}\right) \sin\left(\frac{\pi B \ell \hat{x}}{\Phi_0}\right) \hat{\sigma}_x \quad (3)$$

and making the approximation $\pi B \ell x / \Phi_0 \ll 1$ we find

$$\hat{H} = \omega \hat{a}^\dagger \hat{a} + \frac{1}{2} \omega_0 \hat{\sigma}_z + \lambda_0 (\hat{a}^\dagger + \hat{a}) \hat{\sigma}_x, \quad (4)$$

where the constant coupling $\lambda_0 = -4E_J \cos\left(\frac{\pi\Phi_x}{\Phi_0}\right) \left(\frac{\pi B \ell x_0}{\Phi_0}\right)$ and the effective energy $\omega_0 = 8E_c \left(N_g - \frac{1}{2}\right)$. In the rotating wave approximation the above Hamiltonian results as

$$\hat{H} = \omega \hat{a}^\dagger \hat{a} + \frac{1}{2} \omega_0 \hat{\sigma}_z + \lambda_0 (\hat{\sigma}_+ \hat{a} + \hat{a}^\dagger \hat{\sigma}_-). \quad (5)$$

Next, we will consider a more general scenario by substituting $\omega \rightarrow \omega(t) = \omega + f(t)$ and $\lambda_0 \rightarrow \lambda(t) = \lambda_0 [1 + f(t)/\omega]$ [37, 53]; in addition, we assume the presence of a constant decay rate γ in the CPB, from its excited level to the ground state; ω_0 is the transition frequency of the CPB and λ_0 stands for the CPB-NR coupling. $\hat{\sigma}_\pm$ and $\hat{\sigma}_z$ are the CPB transition and excitation inversion operators, respectively; they act on the Hilbert space of atomic states and satisfy the commutation relations $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$ and $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm \hat{\sigma}_\pm$. As well known, the coupling parameter $\lambda(t)$ is proportional to $\sqrt{V(t)/V(t)}$, where the time dependent quantization volume $V(t)$ takes the form $V(t) = V_0 / [1 + f(t)/\omega]$ [54, 33, 53]. Accordingly, we obtain the new (*non hermitean*) Hamiltonian

$$\hat{H} = \omega(t) \hat{a}^\dagger \hat{a} + \frac{1}{2} \omega_0 \hat{\sigma}_z + \lambda(t) (\hat{\sigma}_+ \hat{a} + \hat{a}^\dagger \hat{\sigma}_-) - i \frac{\gamma}{2} |e\rangle \langle e|. \quad (6)$$

Non hermitean Hamiltonians (NHH) have been largely used in the literature. As some few examples we mention: Ref. [55], where the authors use a NHH and an algorithm to generalize the conventional theory; Ref. [56], using a NHH to get information about entrance and exit channels; Ref. [57], using non hermitean techniques to study canonical transformations in quantum mechanics; Ref. [58], solving quantum master equations in terms of NHH; Ref. [59], using a new approach for NHH to study the spectral density of weak H-bonds involving damping; Ref. [60], studying NHH with real eigenvalues; Ref. [61], using a canonical formulation to study dissipative mechanics exhibiting complex eigenvalues; Ref. [62], studying NHH in non commutative space, and more recently: Ref. [63], studying the optical realization of relativistic NHH; Ref. [33], studying the evolution of entropy of atom-field interaction; Ref. [32], using a damping JC-Model to study entanglement between two atoms, each one inside distinct cavities

3. Solving the CPB-NR system

Now, the state describing our time dependent system can be written as

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} (C_{g,n}(t) |g, n\rangle + C_{e,n}(t) |e, n\rangle). \quad (7)$$

Taking the CPB initially prepared in its excited state $|e\rangle$ and the NR in a superposition of two coherent states, $|\beta\rangle = \eta(|\alpha\rangle + |-\alpha\rangle)$, and expanding each coherent state component in the Fock's basis, i.e., $|\alpha\rangle = \exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} (\alpha^n / \sqrt{n!}) |n\rangle$, we have $|\beta\rangle = \sum_{n=0}^{\infty} F_n |n\rangle$, where $\eta = [2 + 2 \exp(-2\alpha^2)]^{-1/2}$ is the normalization factor. Assuming the NR and CPB decoupled at $t = 0$ and the initial conditions $C_{g,n}(0) = 0$ and $\sum_{n=0}^{\infty} |C_{e,n}(0)|^2 = 1$ we may write the Eq. (7) as

$$|\Psi(0)\rangle = \sum_{n=0}^{\infty} F_n |e, n\rangle. \quad (8)$$

The time dependent Schrödinger equation for the present system is

$$i \frac{d|\Psi(t)\rangle}{dt} = \hat{H} |\Psi(t)\rangle, \quad (9)$$

with the Hamiltonian \hat{H} given in Eq. (6). Substituting Eq.(6) in Eq.(9) we get the (coupled) equations of motion for the probability amplitudes $C_{e,n}(t)$ and $C_{g,n+1}(t)$:

$$\frac{\partial C_{e,n}(t)}{\partial t} = -in\omega(t)C_{e,n}(t) - \frac{i}{2}\omega_0 C_{e,n}(t) - i\lambda(t)\sqrt{n+1}C_{g,n+1}(t) - \frac{\gamma}{2}C_{e,n}(t), \quad (10)$$

$$\frac{\partial C_{g,n+1}(t)}{\partial t} = -i(n+1)\omega(t)C_{g,n+1}(t) + \frac{i}{2}\omega_0 C_{g,n+1}(t) - i\lambda(t)\sqrt{n+1}C_{e,n}(t). \quad (11)$$

The solutions of the coefficients $C_{e,n}(t)$, $C_{g,n+1}(t)$ furnish the quantum dynamical properties of the system, including the CPB-NR entanglement.

For the cases $f(t) = 0$ and $f(t) = \text{const}$, the Eq.(10) and Eq.(11) are exactly soluble. We find, analytically,

$$C_{g,n+1}(t) = (1/\zeta)[-2i\lambda e^{-1/4\delta t} (e^{1/4\zeta t} - e^{-1/4\zeta t}) \sqrt{n+1}F_n], \quad (12)$$

$$C_{e,n}(t) = (1/2\zeta)[ie^{-1/4\delta t} (e^{1/4\zeta t} (i\gamma + 2\omega - i\zeta - 2\omega_0) - e^{-1/4\zeta t} (i\gamma + 2\omega + i\zeta - 2\omega_0)) F_n], \quad (13)$$

where $\delta = \gamma + 2i\omega(1 + 2n)$ and $\zeta = [\gamma(\gamma + 4i(\omega_0 - \omega)) - 4(\omega^2 + \omega_0^2) - 16\lambda^2(1 + n) + 8\omega\omega_0]^{1/2}$. However, when the coupling $f(t)$ is time dependent the solution of this system of equations is found only numerically.

As well known, in the presence of decay rate γ in the CPB the state of the whole CPB-NR system becomes mixed. In this case its description requires the use of the density operator $\hat{\rho}_{CN}$, which describes the entire system. To obtain the reduced density matrix describing the CPB (NR) sub-system we must trace over variables of the NR (CPB) sub-system. For example,

$$\hat{\rho}_{NR} = \text{Tr}_{CPB}(\hat{\rho}_{CN}) = \sum_n \sum_{n'} [C_{e,n}(t)C_{e,n'}^*(t) + C_{g,n}(t)C_{g,n'}^*(t)] |n\rangle \langle n'|. \quad (14)$$

4. Entropy of sub-systems

Recently, researchers have employed several methods to study the dynamical of entanglement [32, 33, 34, 64, 65]. As proved by Phoenix and Knight [66] the von Neumann entropy offers a quantitative measure of disorder of a system and of the purity of a quantum state. Such entropy, defined as $S_{NR(C)} = -\text{Tr}(\hat{\rho}_{N(C)} \ln \hat{\rho}_{N(C)})$, is a measure that is sensible to quantum entanglement of two interacting subsystems. The quantum dynamics described by the Eq. (6) furnishes the CPB-NR entanglement and we will employ the von Neumann quantum entropy as a measure of the degree of entanglement. The entropy S of a quantum system, when composed of two subsystems, obeys a theorem due to Araki and Lieb, which establishes that: $|S_{CPB} - S_{NR}| \leq S \leq S_{CPB} + S_{NR}$; S_{CPB} and S_{NR} standing for the entropies of the subsystems. S stands for the total entropy of CPB-NR system. One immediate consequence of the above inequality is that, if one prepares the entire system in a pure state at $t = 0$, then both components of the whole system have the same entropy for the subsequent time evolution. So, when assuming our system initially in a pure and decoupled state the entropies of the CPB and NR become identical, namely, $S_{CPB}(t) = S_{NR}(t)$. Then, one only needs to calculate the quantum entropy of a subsystem to get its entanglement evolution. We obtain, from the Eqs. (14) and $S_{NR} = -\text{Tr}(\hat{\rho}_{NR} \ln \hat{\rho}_{NR})$,

$$S_{NR}(t) = -[\wedge_{NR}^+(t) \ln(\wedge_{NR}^+(t)) + \wedge_{NR}^-(t) \ln(\wedge_{NR}^-(t))], \quad (15)$$

where,

$$\wedge_{NR}^{\pm}(t) = \frac{1}{2} \left(1 \pm \sqrt{(\langle R_1|R_1 \rangle - \langle R_2|R_2 \rangle)^2 + 4|\langle R_1|R_2 \rangle|^2} \right), \quad (16)$$

with $\langle R_1|R_1 \rangle = \sum_{n=0}^{\infty} |C_{e,n}(t)|^2$, $\langle R_2|R_2 \rangle = \sum_{n=0}^{\infty} |C_{g,n+1}(t)|^2$ and $\langle R_1|R_2 \rangle = \langle R_2|R_1 \rangle^* = \sum_{n=0}^{\infty} C_{e,n+1}^*(t)C_{g,n+1}(t)$.

We can now look at the time evolution of the NR entropy. We will assume the NR subsystem initially in an even ‘‘Schrödinger-cat’’ state. Firstly we consider the resonant case ($f(t) = 0$); the time evolution of the NR entropy with different decay rates γ in the CPB, with $\omega = \omega_0 = 2000\lambda_0$ and the ‘‘cat’’-state with $\alpha = 5$, as shown in Figs. 2(a), 2(b), and 2(c). In an ideal case the CPB decay rate vanishes. As displayed in Fig. 2(a) the maximum value of the entropy of

the NR is close to $\ln 2$. Just after the start of the CPB-NR interaction the entropy of the NR stabilizes at this value by a small interval and then recovers the oscillations as time goes on. The CPB sub-system is stable while standing in its ground state $|g\rangle$; but when lying in its excited state $|e\rangle$, various factors as spontaneous emission among others, imply its decay to the ground state. For a small decay rate (see Fig. 2(b)) the maximum entanglement becomes significant only for large times. However, the increase of the decay rate produces a drastic change on the entanglement (see Fig. 2(c)), with a great reduction in their swings, leading the CPB to its ground state (zero entropy). This effect upon the entropy of the CPB also affects the entropy of the NR (Figs. 2). Secondly, we modify the previous case by including

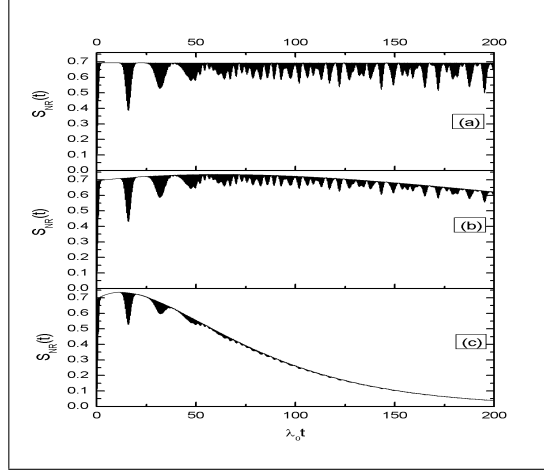


Figure 2: Time evolution of the Entropy when the NR is initially prepared in an even “cat-state”, for different values of the decay rate γ : (a) $\gamma = 0.0\lambda_0$, (b) $\gamma = 0.01\lambda_0$, and (c) $\gamma = 0.05\lambda_0$, with $\alpha = 5$, $\omega = \omega_0 = 2000\lambda_0$, $f(t) = 0$.

the presence of a detuning ($f(t) = \Delta \neq 0$) to verify its influence upon our interacting system. We take the decay rate as $\gamma = 0.05\lambda_0$, with $f(t) = \Delta = \text{const}$ and $\Delta \ll \omega_0, \omega$. As result the entanglement remains for long time as the value of Δ increases, as we see comparing Fig. 3(a) with Fig. 2(c); this event is accompanied by a diminution of the maximum entropy (see Figs 3(a) and 3(b)). When the detuning increases, the CPB transitions decreases (cf. Figs. 9).

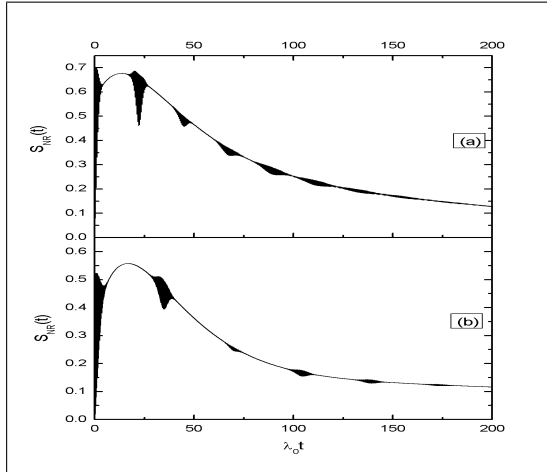


Figure 3: Same as in Fig.2, now for different values of detunings (cf. $f(t) = \Delta = \text{const}$): (a) $\Delta = 10\lambda_0$ and (b) $\Delta = 20\lambda_0$, with $\alpha = 5$, $\omega = \omega_0 = 2000\lambda_0$, $\gamma = 0.05\lambda_0$.

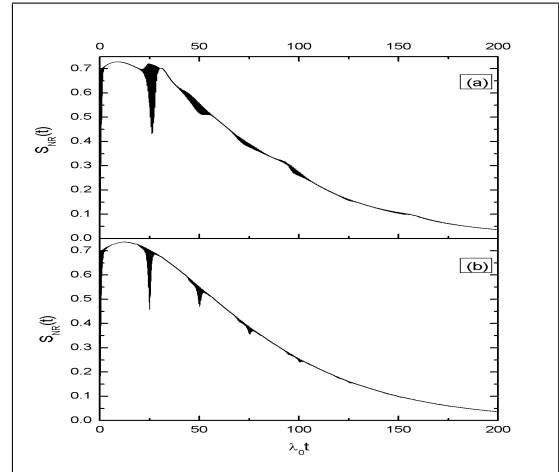


Figure 4: Same as in Figs. 2 and 3, for time-dependent detunings (cf. $f(t) = c \sin(\omega t)$): with (a) $c = 20\lambda_0$ e $\omega = 0.1\lambda_0$, and (b) $c = 20\lambda_0$ e $\omega = 0.5\lambda_0$

Thirdly, we extend the detuning to the time dependent case, assuming $f(t) = c \sin(\omega t)$, where c and ω are

parameters of amplitude and frequency modulation of the NR with the condition $\omega' < c \ll \omega_0, \omega$. Comparing the Fig. 4(a) with Fig. 3(b) we see that the sinusoidal modulation does not favor the entanglement for long time. However, the frequency modulation turns the maxima of entanglement greater (see Fig. 4). Comparing Fig. 4(a) with 4(b) we see that when the frequency ω' grows the oscillations of the entropy decrease.

5. Power Spectrum of the Entropy

To get a better understanding of the entropy we have considered its power spectrum (**PS**). It consists of a frequency-dependent function, being real, positive, and constructed from the following Fourier transform [54]

$$PS(\varpi) = \frac{1}{\pi} \int_0^{\tau_{\max}} S_{NR}(t) \exp(i\varpi t) dt, \quad (17)$$

where $\tau_{\max} = \lambda_0 t_{\max}$ stands for the maximum interaction interval in the plot $S_{NR}(t)$ versus $\lambda_0 t$.

The entropy PS is obtained from the above equation and plotted in Figs. 2(a), 2(b) and 2(c). As expected, the amplitude of oscillations of this PS is reduced in the presence of growing decay rates (see Figs. 5(a), 5(b) and 5(c)); when we add a constant detuning ($f(t) = \Delta \neq 0$) and a decay rate $\gamma = 0.05\lambda_0$ we see in Figs. 6(a) and 6(b) the maximum value of the PS increasing when Δ also increases. In the case $f(t) = c \sin(\omega' t)$ the frequency of entropy PS is smoothly attenuated, with a peak around $\Omega = 0.1$, as shown in Fig. 7(a). When the frequency ω' increases the entropy PS is rapidly attenuated (cf. Figs. 7(a) and 7(b)).

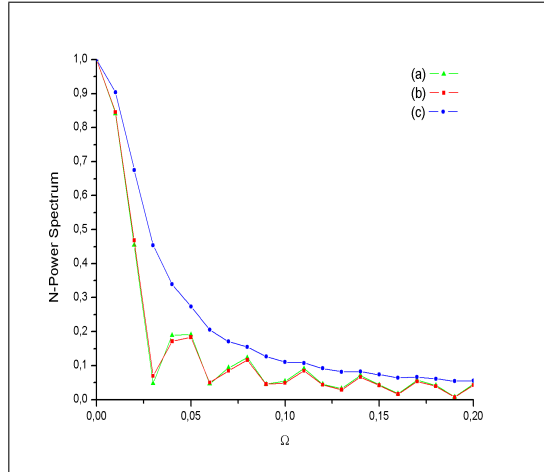


Figure 5: Time evolution of the Normalized Power Spectrum when the NR is initially prepared in “cat-state”, concerning with the Entropies shown in the plots of Fig. 2: the plot 5(i) refers to the plot 2(i), $i = a, b, c$.

6. Excitation Inversion of the CPB

The CPB excitation inversion, $I_{CPB}(t)$, is an important observable of two level systems. It is defined as the difference of the probabilities of finding this system in the excited and in the ground state; for the CPB it reads,

$$I_{CPB}(t) = \sum_{n=0}^{\infty} \left[|C_{e,n}(t)|^2 - |C_{g,n+1}(t)|^2 \right]. \quad (18)$$

The Eq. (18) allows us to look at the time evolution of the CPB excitation inversion. First, we assume the resonant case ($f(t) = 0$), for different values of the decay rate γ , with $\alpha = 5$ and $\omega = \omega_0 = 2000\lambda_0$ as in Fig. 8. Figs. 8(a), (b) and (c) exhibit identical collapse and revival, but with different amplitudes: the higher the decay rate, the lower the amplitude of oscillations of CPB excitation inversion. However, in the presence of a fixed detuning, with

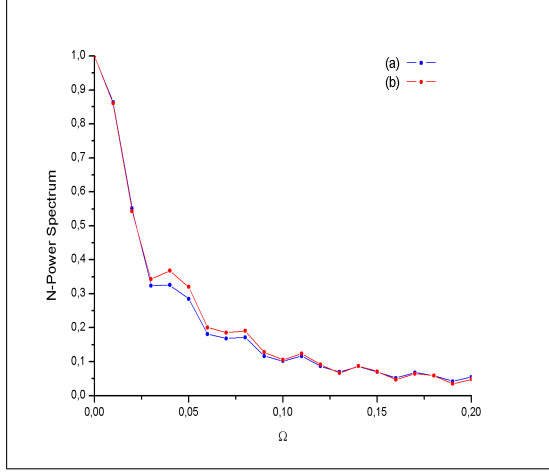


Figure 6: Same as in Fig. 5, concerning with the Entropies shown in the plots of Fig. 3: the plot 6(i) refers to the plot 3(i), $i = a, b$.

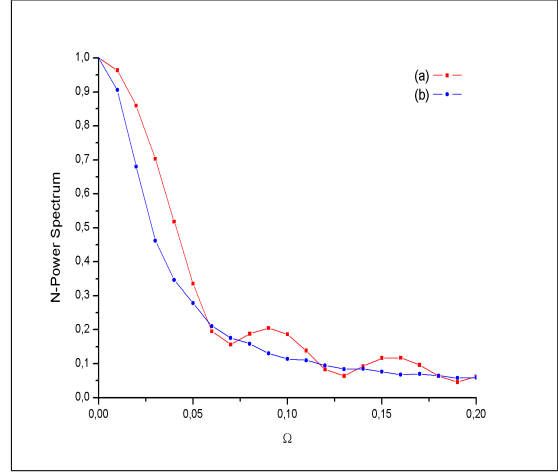


Figure 7: Same as in Figs. 5 and 6, concerning with the Entropies shown in the plots of Fig. 4: the plot 7(i) refers to the plot 4(i), $i = a, b$.

$f(t) = \Delta = \text{const}$ and $\Delta \ll \omega_0, \omega$, we see that CPB excitation inversion in Fig. 9(a) occurs only inside the interval $\tau = \lambda_0 t \in (15, 30)$; differently, in Fig. 9(b) this event occurs inside the interval $\tau \in (50, 75)$, with amplitude smaller than that in Fig. 9(a); this is the effect caused by a constant detuning upon the CPB excitation inversion.

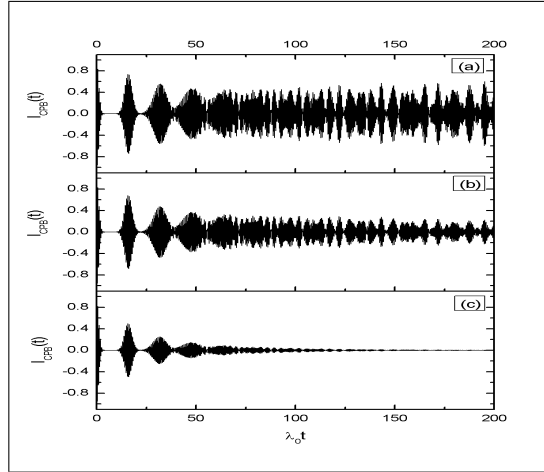


Figure 8: Time evolution of the CPB Excitation Inversion with the NR initially prepared in the even “cat-state”, for various values of the decay rate: (a) $\gamma = 0.0\lambda_0$ (b) $\gamma = 0.01\lambda_0$, and (c) $\gamma = 0.05\lambda_0$, with $\alpha = 5$, $\omega = \omega_0 = 2000\lambda_0$, and $f(t) = 0$ (resonance).

For a time dependent detuning, $f(t) = c \sin(\omega t)$, the frequency of the CPB excitation inversion accompanies the frequency ω , as shown in Figs. 10. When we compare the Fig. 10(b) with Fig. 8(c), we see: if ω increases, the interval of collapse of the excitation inversion also increases. However, looking at Figs. 10(a), 10(b) we see that the increasing of ω as in Fig. 10(b) introduces equally spaced collapse intervals, accompanied by revivals modulated by the parameter ω . Now, we compare the CPB excitation inversion with constant detuning ($f(t) = \Delta = \text{const}$) and with a time dependent detuning ($f(t) = c \sin(\omega t)$): looking at Figs. 9 we see the plots of excitation inversion showing neither collapses nor revivals, with exceptions of small regions exhibiting excitation inversion (Fig.9(a)); in Fig 9(b) only a single such region appears. However, when considering a time dependent detuning (Fig. 10(b)), it nicely restitutes those collapses and revivals that appear in the resonant case (cf. Fig. 8(c)).

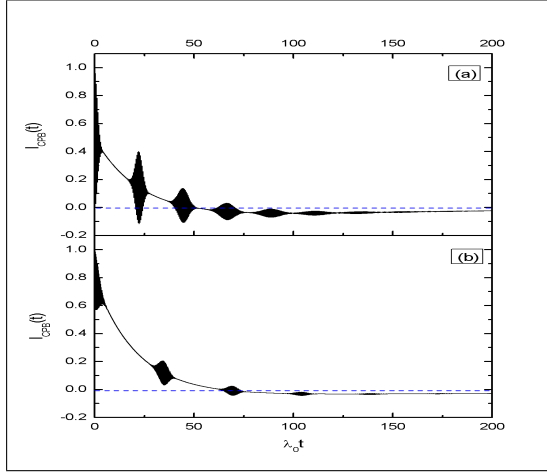


Figure 9: Same as in Fig. 8, for different values of detuning (cf. $f(t) = \Delta = \text{const}$): (a) $\Delta = 10\lambda_0$ and (b) $\Delta = 20\lambda_0$, with $\alpha = 5$, $\omega = \omega_0 = 2000\lambda_0$, $\gamma = 0.05\lambda_0$.

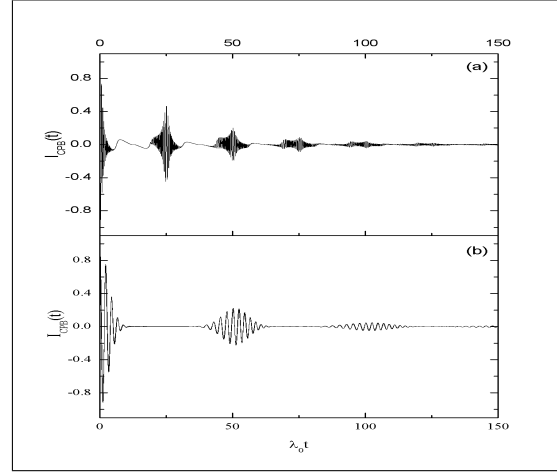


Figure 10: Same as in Figs. 8 and 9, for time-dependent detunings (cf. $f(t) = c \sin(\omega t)$): (a) $c = 20\lambda_0$, $\omega t = 0.5\lambda_0$, (b) $c = 60\lambda_0$, $\omega t = 20\lambda_0$, with $\alpha = 5$, $\omega = \omega_0 = 2000\lambda_0$, $\gamma = 0.05\lambda_0$.

7. Conclusion

We have considered a Hamiltonian model for a CPB-NR interacting system to study Entropy, its Power Spectrum and the CPB Excitation Inversion. These properties characterize the entangled state that describes this coupled system for various values of the parameters involved. We have included dissipation and assumed the NR initially in a Schrödinger “cat”-state and the CPB in excited state. We have also considered the following scenarios: (i) both subsystems in resonance (detuning $f = 0$); (ii) off-resonance, with a constant detuning ($f = \Delta \neq 0$), and (iii) with a time dependent detuning ($f(t) = c \sin(\omega t)$). The results were discussed in the previous section. Concerning with the entropy we see that when the NR is initially in a Schrödinger “cat”-state, the entropy lasts longer than in an atom-field system, with the field initially in a coherent state (cf. Ref. [33]). Concerning the Excitation Inversion, an interesting result emerges: although the presence of a constant detuning destroys the collapse and revivals of the excitation inversion, these effects are restituted by the action of convenient time dependent detunings - even in the presence of damping. It is also worth emphasizing that the presence of an external force upon the NR changes the magnetic flux Φ_e (cf. Fig. 1), which provides the control of the parameters $\omega(t)$ and $\lambda(t)$.

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